Problem 1.15

The current entering the positive terminal of a device is $i(t) = 6e^{-2t}$ mA and the voltage across the device is $v(t) = 10di/dt$ V.

(a) Find the charge delivered to the device between $t = 0$ and $t = 2$ s.

(b) Calculate the power absorbed.

(c) Determine the energy absorbed in 3 s.

[TYPO: This should be millivolts (mV).]

Solution

Part (a)

Begin with the basic definition of current and integrate both sides with respect to time from 0 to 2.

$$\frac{dq}{dt} = i(t)$$

$$\int_0^2 \frac{dq}{dt}\,dt = \int_0^2 i(t)\,dt$$

$$\begin{aligned} q(2) - q(0) &= \int_0^2 6e^{-2t}\,dt \text{ mA} \\ &= (-3e^{-2t})\Big|_0^2 \text{ mC} \\ &= -3(e^{-4} - e^0) \text{ mC} \\ &\approx 2.95 \text{ mC} \end{aligned}$$

This is the charge delivered to the device between $t = 0$ and $t = 2$ s.

Part (b)

The power that the device absorbs is

$$p(t) = v(t)i(t) = \left[10\frac{d}{dt}(6e^{-2t}) \text{ mV}\right](6e^{-2t} \text{ mA}) = [10(-12e^{-2t})](6e^{-2t})\ \mu\text{W} = -720e^{-4t}\ \mu\text{W}.$$

Part (c)

Integrate the power with respect to time from $t = 0$ to $t = 3$ s to obtain the energy absorbed during this interval.

$$\begin{aligned} W = \int_0^3 p(t)\,dt = \int_0^3 (-720e^{-4t})\,dt\ \mu\text{W} = -720\int_0^3 e^{-4t}\,dt\ \mu\text{W} &= 180e^{-4t}\Big|_0^3\ \mu\text{J} \\ &= 180(e^{-12} - e^0)\ \mu\text{J} \\ &\approx -180\ \mu\text{J} \end{aligned}$$